Assessment of the South African anchovy resource using data from 1984 – 2010: variability in natural mortality

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Introduction

Initial results of the updated assessment of the South African anchovy resource, using data from 1984 to 2010, were presented by de Moor and Butterworth (2011). This work included the use of a random effects model for adult natural mortality with a fixed standard deviation of 0.2 and 0.5. This approach has now been further extended such that annual autocorrelation can be included and the associated parameters can be estimated, rather than fixed. Similar testing on the use of random effects models for juvenile natural mortality has also been undertaken.

Population Dynamics Model

The population dynamics model used for the South African anchovy resource is detailed in Appendix A. The data used in this assessment are listed in de Moor *et al.* (2011), with a subsequent update to the weights-at-age. The prior distributions for the estimated parameters were chosen to be relatively uninformative.

Results

Random effects on adult natural mortality $\left(M^{A}_{ad,y}\right)$ were modelled first with no autocorrelation between annual adult natural mortality ($p=0$ in equation (A.8)) and then with autocorrelation included. Table 1 shows the contributions to the joint posterior of the likelihoods for the different sets of data and the prior distributions. The best overall posterior value (i.e. the posterior mode) is obtained when both σ_{ad} and p are estimated, with values of 0.26 and 0.43 at this joint posterior mode.

The population model fits to the time series of abundance estimates of November 1+ biomass, DEPM estimates of spawner biomass, May recruitment and proportion-at-age 1 in November are shown in Figures 1 to 4 respectively for four cases: i) no random effects on $M^{A}_{ad,y}$ ($\varepsilon^{ad}_{y}=0$, $p=0$), ii) random effects on $M^{A}_{ad,y}$ with no autocorrelation ($\sigma_{ad}=0.34$, $p=0$), iii) random effects on $M^{A}_{ad,y}$ with autocorrelation (fixed $\sigma_{ad}=0.15$, $p=0.62$), and iv) random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad}=0.26$, $p=0.43$).

Although the model projected posterior mode estimates of 1+ biomass and May recruitment in 2010 still fall near the extremes of the 95% PI due to the model struggling to match a sharp decrease in the 1+ biomass after a

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relatively good recruitment, the fits are an improvement over previous results. The fit to the May recruit data is substantially improved with the inclusion of random effects (Table 1, Figure 3).

The fit to proportion-at-age 1 is also substantially improved with the inclusion of random effects (Table 1, Figure 4). High σ_{ad} values result in excellent fits to the proportion-at-age 1 data, but with unrealistically high deviations from the median adult natural mortality. The more realistic ranges of $M_{ad,y}^{A}$ shown in Figure 5 still correspond to good fits to the proportion-at-age 1 data (Table 1, Figure 4). When both σ_{ad} and p are estimated, the range of $M_{ad,y}^{A}$ is [0.62, 1.89]. When σ_{ad} is fixed at a low value (0.15), autocorrelation is estimated to be higher but the range of values is reduced to [0.83,1.39] (Table 1, Figure 5).

The estimated Beverton-Holt stock recruitment curve is plotted in Figure 6. The parameter values estimated at the posterior mode are given in Table 2.

The model was similarly extended to also include a random effects model on juvenile natural mortality. This showed that there was no statistical justification for including such temporal variability in juvenile natural mortality.

Summary and Future Work

The use of a random effects model for adult natural mortality has resolved the former problem of perceived trends in the residuals from the model fit to May recruitment and the November proportion-at-age 1 data (de Moor and Butterworth 2011). The results presented in this document show that autocorrelation can and should be included in these random effects. The authors propose that the option where both the standard deviation, σ_{ad}, and the autocorrelation coefficient, p, are estimated be taken forward in the base case model. The posterior distributions estimated for these two parameters will then provide the information necessary to model future changes in adult natural mortality when simulation testing OMP-12. Robustness tests to the base case anchovy model will then include the case of no autocorrelation (ii above) and the case of a fixed small standard deviation with strong autocorrelation (iii above).

Juvenile natural mortality will be treated as time-invariant, with model sensitivity to alternative values to be tested using robustness tests.

Having now finalised how adult and juvenile natural mortality will be modelled in the anchovy assessment, alternative stock recruitment relationships (including ones that admit change over time) will need to be retested. Alternative median juvenile and adult natural mortality values will also need to be tested before a base case model can be chosen. Markov Chain Monte Carlo will be used to simulate posterior distributions of key model parameters for use in simulation testing OMP-12.

References

de Moor, C.L., and Butterworth, D.S. 2011. Assessment of the South African anchovy resource using data from 1984 – 2010: initial results. DEAT: Branch FisheriesDocument FISHERIES/2011/SWG-PEL/06. 11pp.

de Moor, C.L., van der Westhuizen, J.J., Durholtz D. and Coetzee, J. 2011a. A Record of the Generation of Data Used in the 2011 Sardine and Anchovy Assessments. Unpublished Department of Agriculture, Forestry and Fisheries Document FISHEREIS/2011/SWG-PEL/04. 31pp.

Table 1. The individual contributions to the negative log posterior at the mode for alternative values of σ_{ad} and p. **Bold** values represent those that are fixed rather than estimated using the prior distributions $\sigma_{ad} \sim U(0.1,0.5)$ and $p \sim U(0,1)$. The row in *italics* represents the best overall fit of the model.

σ_{ad}	p	lnPosterior	$\ln L_{Nov}$	$\ln L_{Egg}$	$\ln L_{Recruit}$	$\ln L_{Prop1}$	$\ln Prior_{Recruit}$	$\ln Prior_{Madult}$
0.00	**0.00**	66.59	-3.95	7.72	10.97	29.03	22.82	-
0.20	**0.00**	32.90	-3.75	7.81	5.14	4.70	20.17	-1.17
0.30	**0.00**	30.58	-8.00	6.84	4.84	-0.57	20.48	6.99
0.34	**0.00**	30.44	-8.86	6.62	4.77	-1.52	20.57	8.86
0.40	**0.00**	30.77	-10.00	6.33	4.66	-2.62	20.72	11.68
0.50	**0.00**	32.14	-11.17	6.03	4.52	-3.55	20.90	15.39
0.15	**0.25**	30.33	-3.62	7.93	5.22	15.97	20.71	-15.88
0.15	**0.5**	29.07	-3.19	8.09	4.70	13.75	20.63	-14.90
0.20	**0.25**	29.29	-4.08	7.74	4.95	2.70	20.14	-2.16
0.20	**0.5**	28.11	-4.18	7.77	4.84	2.56	20.38	-3.25
0.15	0.62	28.96	-3.94	7.98	4.75	15.06	20.96	-15.84
0.20	0.47	28.09	-4.17	7.77	4.85	2.45	20.33	-3.15
0.26	*0.43*	*27.51*	*-6.69*	*7.10*	*4.89*	*-0.54*	*20.52*	*2.22*
0.30	0.40	27.66	-7.71	6.81	4.88	-1.57	20.60	4.64
0.40	0.34	29.05	-9.62	6.29	4.76	-3.08	20.78	9.93
0.50	0.29	31.08	-10.87	5.99	4.60	-3.77	20.94	14.19

Table 2. Key parameter values estimated at the joint posterior mode together with key model outputs. All parameters are defined in the Appendix. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tons.

	Option			
	i)	ii)	iii)	iv)
σ_{ad}	NA	0.34	**0.15**	0.26
p	NA	**0.00**	0.62	0.43
$N^A_{1983,0}$	155.6	160.1	162.8	161.4
$N^A_{1983,1}$	141.3	152.9	142.1	141.9
$N^A_{1983,2}$	0.005	0.005	0.005	0.005
$N^A_{1983,3}$	0.005	0.005	0.005	0.005
k^A_N	1.193	1.115	1.140	1.129
k^A_r	1.072	0.896	0.914	0.890
k^A_r / k^A_N	0.899	0.803	0.802	0.788
k^A_p	0.968	0.962	0.942	0.946
$\left(\sigma^A_p\right)^2$	0.503	0.090	0.179	0.090
$\left(\lambda^A_N\right)^2$	**0.000**	**0.000**	**0.000**	**0.000**
$\left(\lambda^A_r\right)^2$	0.102	0.051	0.050	0.052
$\hat{B}^A_{2010,N}$	2343.7	2197.7	2712.6	2197.0
$\overline{B}^A_{Nov}$ [1]	1100.5	1190.9	1150.7	1179.3
K^A	3519.3	9897.1	8333.7	9897.1
h^A	0.351	0.294	0.319	0.315
σ^A_r	0.582	0.534	0.542	0.533
η^A_{2009}	-0.491	-0.622	-0.591	-0.571
s^A_{cor}	0.215	0.167	0.177	0.189

[1] OMP-04 and OMP-08 were developed using Risk defined as "the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years".

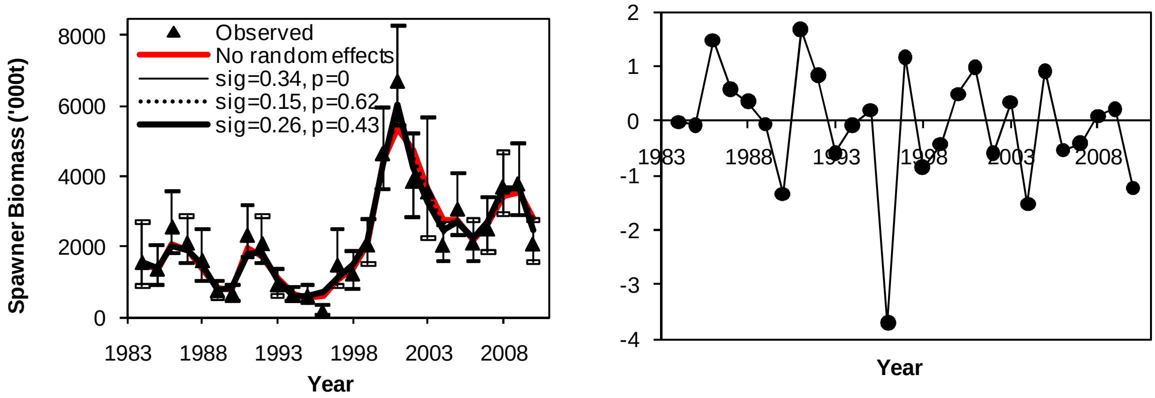

Figure 1. Acoustic survey results and model estimates for November anchovy spawner biomass from 1984 to 2010 using i) no random effects on $M^{A}_{ad,y}$ ($\varepsilon^{ad}_{y} = 0$, $p = 0$), ii) random effects on $M^{A}_{ad,y}$ with no autocorrelation ($\sigma_{ad} = 0.34$, $p = 0$), iii) random effects on $M^{A}_{ad,y}$ with autocorrelation (fixed $\sigma_{ad} = 0.15$, $p = 0.62$), and iv) random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad} = 0.26$, $p = 0.43$). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit to iv) are given in the right hand plot.

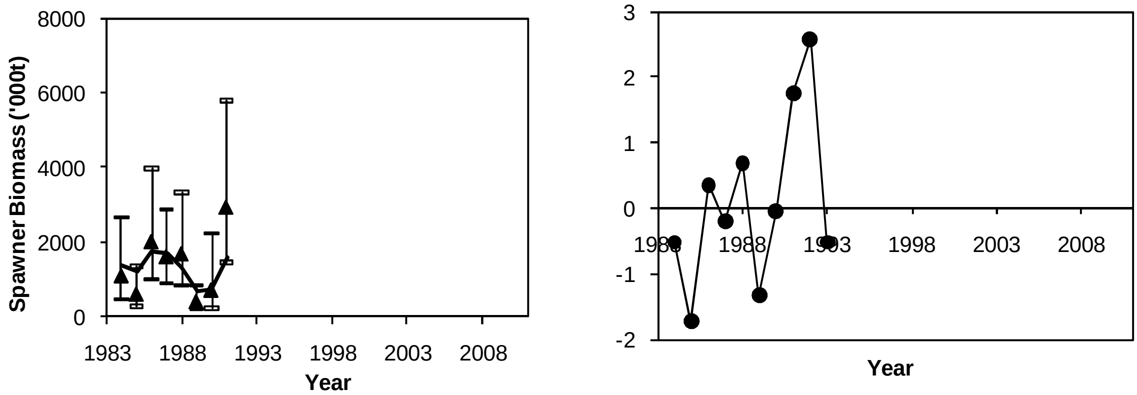

Figure 2. Egg survey results and model estimates for November anchovy spawner biomass from 1984 to 1991 for random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad} = 0.26$, $p = 0.43$). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

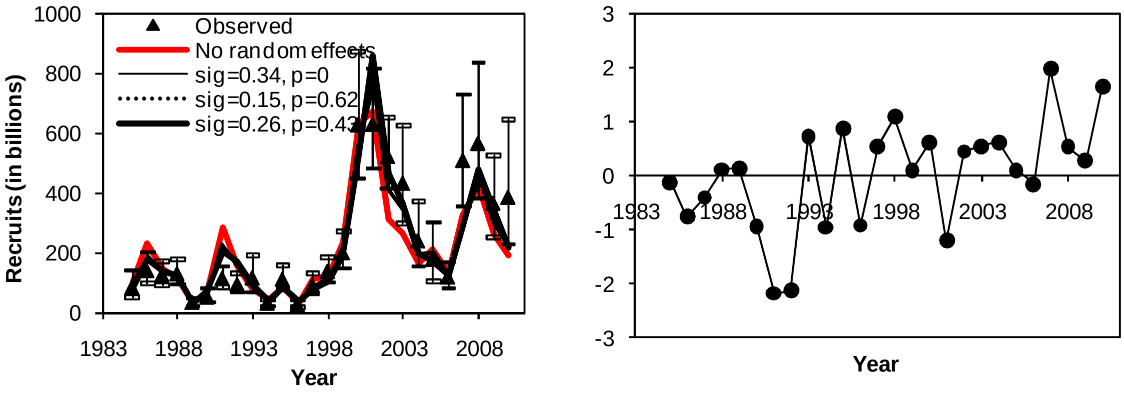

Figure 3. Acoustic survey results and model estimates for anchovy recruitment numbers from May 1985 to May 2010 using i) no random effects on $M^{A}_{ad,y}$ ($\varepsilon^{ad}_{y} = 0$, $p = 0$), ii) random effects on $M^{A}_{ad,y}$ with no autocorrelation ($\sigma_{ad} = 0.34$, $p = 0$), iii) random effects on $M^{A}_{ad,y}$ with autocorrelation (fixed $\sigma_{ad} = 0.15$, $p = 0.62$), and iv) random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad} = 0.26$, $p = 0.43$). The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit to iv) are given in the right hand plot.

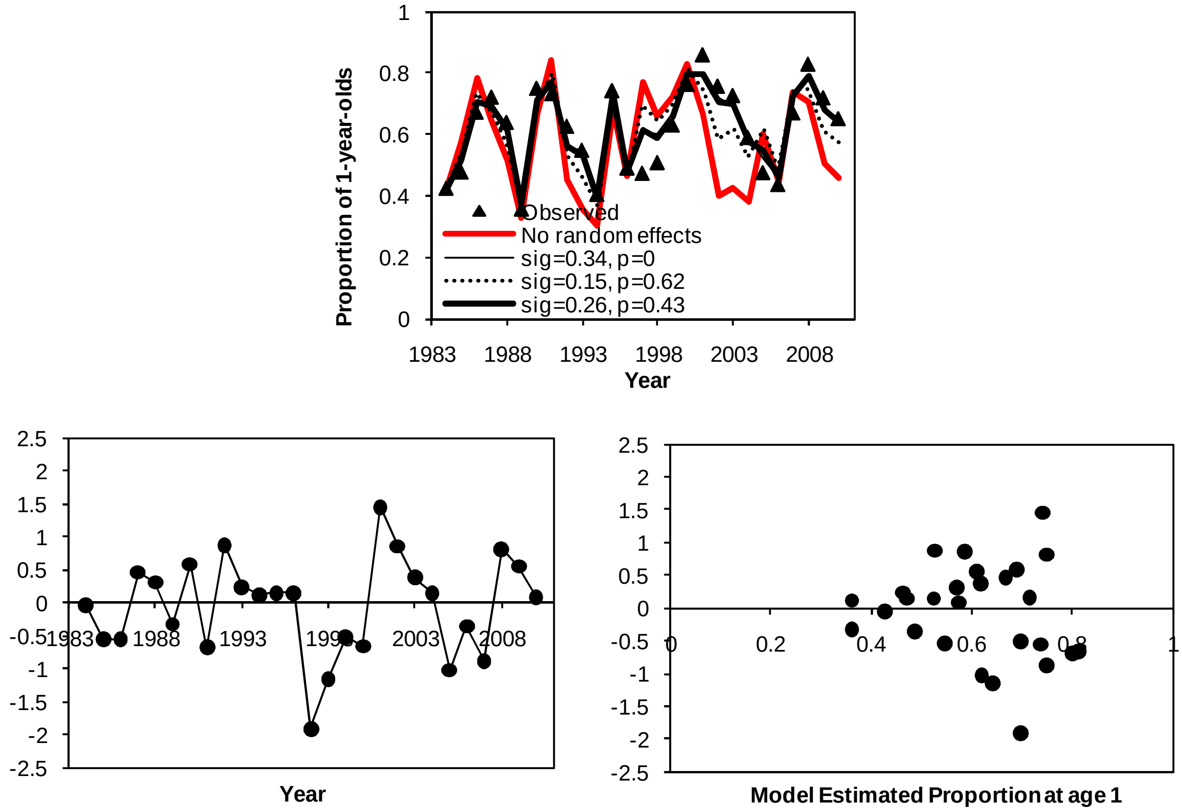

Figure 4. Acoustic survey results and model estimates for proportions of 1-year-olds in the November survey from 1984 to 2010 using i) no random effects on $M^{A}_{ad,y}$ ($\varepsilon^{ad}_{y} = 0$, $p = 0$), ii) random effects on $M^{A}_{ad,y}$ with no autocorrelation ($\sigma_{ad} = 0.34$, $p = 0$), iii) random effects on $M^{A}_{ad,y}$ with autocorrelation (fixed $\sigma_{ad} = 0.15$, $p = 0.62$), and iv) random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad} = 0.26$, $p = 0.43$). The standardised residuals from the fit to iv) are given in the lower plots, against year and against model estimates of proportions at age 1.

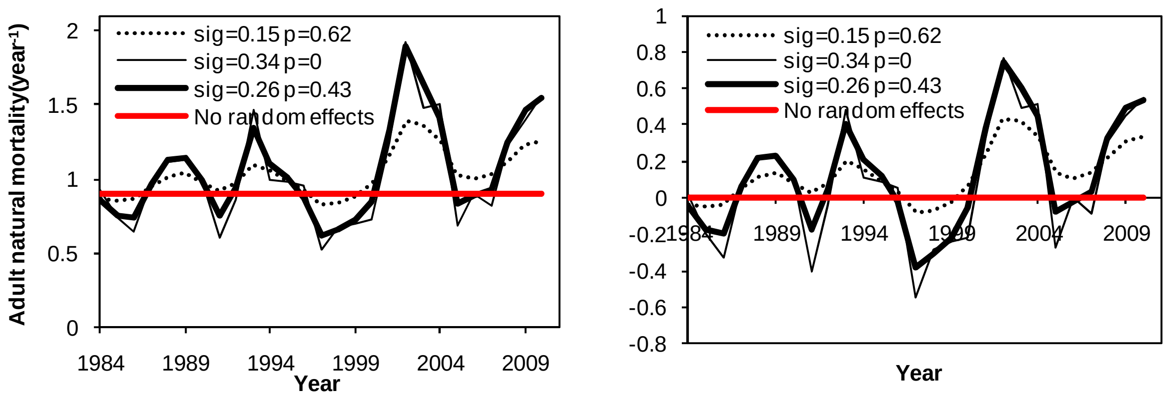

Figure 5. Model estimated annual adult natural mortality for i) ii) iii) and iv). The random effects are plotted in the right hand panel.

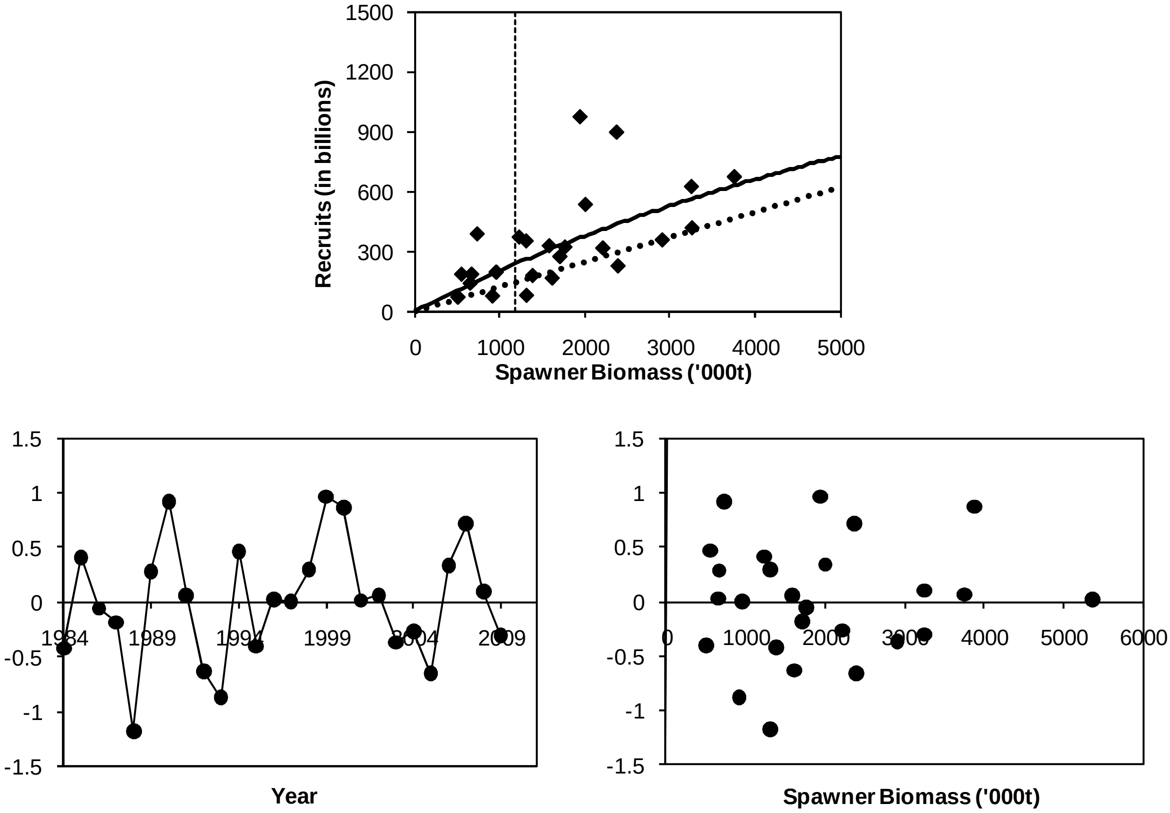

Figure 6. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2010, with the Beverton Holt stock-recruit relationship for random effects on $M^{A}_{ad,y}$ with autocorrelation (estimated $\sigma_{ad} = 0.26$, $p = 0.43$). The vertical thin dashed line indicates the average 1984 to 1999 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, against year and against spawner biomass.

APPENDIX: Bayesian Assessment Model for the South African Anchovy Resource

Model Assumptions

1) All fish have a theoretical birthdate of 1 November.
2) Anchovy spawn for the first time (and are called adult anchovy) when they turn one year old.
3) A plus group of age 4 is used, thus assuming that natural mortality is the same for age 4 and older ages.
4) Two acoustic surveys are held each year: the first takes place in November and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile anchovy.
5) The November acoustic survey provides a relative index of abundance of unknown bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The egg survey observations (derived from data collected during the earlier November surveys) provide absolute indices of abundance.
8) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
9) Pulse fishing occurs five months after 1 November for 1-year-old anchovy; for 0-year-old anchovy this occurs 7½ months after 1 November prior to 1999, and 8½ months after 1 November from 1999 onwards; these two ages (0 and 1) are the only ages targeted by the fishery.
10) Catches are measured without error. (Selectivity of age 0 and age 1 anchovy varies from year to year. This would prove problematic were model predicted catch to be estimated and fitted to observed catch, but here the observed catches-at-age are directly incorporated into the dynamics.)
11) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

The basic dynamic equations for anchovy are as follows, where $y_n = 2010$.

Numbers-at-age at 1 November

$$N^A_{y,1} = (N^A_{y-1,0}e^{-(7.5)M^A_{j,y}/12} - C^A_{y,0})e^{-(4.5)M^A_{j,y}/12} \qquad y = 1984, \ldots, 1998$$

$$N^A_{y,1} = (N^A_{y-1,0}e^{-(8.5)M^A_{j,y}/12} - C^A_{y,0})e^{-(3.5)M^A_{j,y}/12} \qquad y = 1999, \ldots, y_n$$

$$N^A_{y,2} = (N^A_{y-1,1}e^{-5M^A_{ad,y}/12} - C^A_{y,1})e^{-7M^A_{ad,y}/12} \qquad y = 1984, \ldots, y_n$$

$$N^A_{y,3} = N^A_{y-1,2}e^{-M^A_{ad,y}} \qquad y = 1984, \ldots, y_n$$

$$N^A_{y,4+} = N^A_{y-1,3}e^{-M^A_{ad,y}} \qquad y = 1984$$

$$N^A_{y,4+} = N^A_{y-1,3}e^{-M^A_{ad,y}} + N^A_{y-1,4+}e^{-M^A_{ad,y}} \qquad y = 1985, \ldots, y_n \qquad \text{(A.1)}$$

where

$N^A_{y,a}$ is the number (in billions) of anchovy of age *a* at the beginning of November in year *y*;

$C_{y,a}^{A}$ is the number (in billions) of anchovy of age *a* caught from 1 November in year $y-1$ to 31 October in year y;

$M_{j,y}^{A}$ is the annual natural mortality (in year^{-1}) of juvenile anchovy (i.e. fish of age 0) in year y; and

$M_{ad,y}^{A}$ is the annual natural mortality (in year^{-1}) of adult anchovy (i.e. fish of age 1+) in year y.

Biomass associated with the November survey

$$\hat{B}_{y,N}^{A} = \sum_{a=1}^{4+} N_{y,a}^{A} w_{y,a}^{A} \qquad y = 1984, \ldots, y_n \qquad \text{(A.2)}$$

where:

$\hat{B}_{y,N}^{A}$ is the biomass (in thousand tons) of adult anchovy at the beginning of November in year *y*, which are taken to be associated with the November survey; and

$w_{y,a}^{A}$ is the mean mass (in grams) of anchovy of age *a* sampled during the November survey of year *y*.

Anchovy are assumed to mature at age 1 and thus the spawning stock biomass is:

$$SSB_{y,N}^{A} = \sum_{a=1}^{4+} N_{y,a}^{A} w_{y,a}^{A} \qquad y = 1984, \ldots, y_n \qquad \text{(A.3)}$$

Recruitment

Recruitment at the beginning of November is assumed to fluctuate lognormally about a stock-recruitment curve:

$$N_{y,0}^{A} = f\left(SSB_{y,N}^{A}\right) e^{\varepsilon_y^A} \qquad y = 1984, \ldots, y_{n-1} \qquad \text{(A.4)}$$

where

$f\left(SSB_{y,N}^{A}\right) = \dfrac{\alpha^A SSB_{y,N}^{A}}{\beta^A + SSB_{y,N}^{A}}$ for the Beverton Holt curve, with $\alpha^A = \dfrac{4h^A}{5h^A - 1}\dfrac{K^A}{X}$, $\beta^A = \dfrac{K^A\left(1 - h^A\right)}{5h^A - 1}$ and

$X = \sum_{a=1}^{3} \overline{w}_a^A e^{-M_j^A - (a-1)M_{ad}^A} + \overline{w}_{4+} e^{-M_j^A - 3M_{ad}^A} \dfrac{1}{1 - e^{-M_{ad}^A}}$, and

h^A is the steepness associated with the stock-recruitment curve

K^A is the carrying capacity

ε_y^A is the annual lognormal deviation of anchovy recruitment.

Number of recruits at the time of the recruit survey

The following equation projects $N_{y,0}^{A}$ to the start of the recruit survey, taking natural and fishing mortality into account, and assuming pulse fishing of juveniles at 1 May (based on historic data).

$$\hat{N}_{y,r}^{A} = (N_{y-1,0}^{A} e^{-0.5M_j^A} - C_{y,0bs}^{A}) e^{-t_y^A \times M_j^A / 12} \qquad y = 1985, \ldots, y_n \qquad \text{(A.5)}$$

where

$\hat{N}^A_{y,r}$ is the number (in billions) of juvenile anchovy at the time of the recruit survey in year *y*;

$C^A_{y,0bs}$ is the number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year *y*;

t^A_y is the time lapsed (in months) between 1 May and the start of the recruit survey that provided the estimate $N^A_{y,rec}$ in year *y*.

Proportions of 1-year-olds associated with November survey

$$\hat{p}^A_{y,1} = \frac{N^A_{y,1}}{\sum_{a=1}^{4+} N^A_{y,a}} \qquad y = 1984, \ldots, y_n \qquad \text{(A.6)}$$

where

$\hat{p}^A_{y,1}$ is the proportion of 1-year-old anchovy at the beginning of November in year *y*, which is taken to be associated with the November survey.

Fitting the Model to Observed Data (Likelihood)

The observations are assumed to be log-normally distributed, and sampling CVs (squared) of the untransformed survey observations are used to approximate the "sampling" component of the total variance of the corresponding log-distributions. The proportions of 1-year-olds are first logit-transformed before being used in the likelihood[2]. Thus we have:

$$\begin{aligned}
-\ln L = {} & \tfrac{1}{2}\sum_{y=1984}^{yn}\left\{\frac{\left(\ln B^A_{y,N} - \ln(k^A_N \hat{B}^A_{y,N})\right)^2}{(\sigma^A_{y,N})^2 + (\lambda^A_N)^2} + \ln\left[2\pi\left((\sigma^A_{y,N})^2 + (\lambda^A_N)^2\right)\right]\right\} \\
& + \tfrac{1}{2}\sum_{y=1984}^{1991}\left\{\frac{\left(\ln B^A_{y,egg} - \ln(k^A_g \hat{B}^A_{y,N})\right)^2}{(\sigma^A_{y,egg})^2} + \ln\left[2\pi(\sigma^A_{y,egg})^2\right]\right\} \\
& + \tfrac{1}{2}\sum_{y=1985}^{yn}\left\{\frac{\left(\ln N^A_{y,r} - \ln(k^A_r \hat{N}^A_{y,r})\right)^2}{(\sigma^A_{y,r})^2 + (\lambda^A_r)^2} + \ln\left[2\pi\left((\sigma^A_{y,r})^2 + (\lambda^A_r)^2\right)\right]\right\} \qquad \text{(A.7)} \\
& + \frac{1}{2}\sum_{y=1984}^{yn}\left\{\frac{\left(\ln\left(p^A_{y,1}/\left(1-p^A_{y,1}\right)\right) - \ln\left(k^A_p \hat{p}^A_{y,1}/\left(1-k^A_p \hat{p}^A_{y,1}\right)\right)\right)^2}{\left(\sigma^A_p\right)^2} + \ln\left[2\pi\left(\sigma^A_p\right)^2\right]\right\}
\end{aligned}$$

where

$B^A_{y,N}$ is the acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year *y*, with associated CV $\sigma^A_{y,N}$ and constant of proportionality (multiplicative bias) k^A_N;

[2] This transformation proved adequate, resulting in no heteroscedasticity in the residuals of the logit transformation.

$B^A_{y,egg}$ is the egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year *y*, with associated CV $\sigma^A_{y,egg}$ and constant of proportionality k^A_g;

$N^A_{y,r}$ is the acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year *y*, with associated CV $\sigma^A_{y,r}$ and constant of proportionality k^A_r;

$p^A_{y,1}$ is an estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year *y*,. For the base case assessment an average Prosch age length key is used to derive these proportions;

k^A_p is a multiplicative bias associated with the proportion of 1-year-olds in the November survey;

$(\lambda^A_{N/r})^2$ is the additional variance (over and above the survey sampling CV $\sigma^A_{y,N/r}$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

σ^A_p is the standard deviation associated with the proportion of 1-year-olds in the November survey, which is estimated in the fitting procedure by:

$$\sigma^A_p = \sqrt{\sum_{y=1984}^{2006} \left[\ln\left(p^A_{y,1} \big/ \left(1 - p^A_{y,1}\right)\right) - \ln\left(k^A_p \hat{p}^A_{y,1} \big/ \left(1 - k^A_p \hat{p}^A_{y,1}\right)\right)\right]^2 \Big/ \sum_{y=1984}^{2006} 1}$$

Fixed Parameters

Three parameters are fixed externally in this assessment (see main text for reasons and for variations for robustness tests):

$M^A_{j,y} = 0.9$ for all years, $\left(\lambda^A_N\right)^2 = 0$, and $k^A_g = 1$, as the egg survey estimates of abundance are assumed to be absolute.

Adult natural mortality varies around 0.9 as follows

$$M^A_{ad,y} = 0.9e^{\varepsilon_{ad,y}} \text{ with } \varepsilon^{ad}_y = p\varepsilon^{ad}_{y-1} + \sqrt{1-p^2}\,\eta^{ad}_y \qquad \text{(A.8)}$$

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve:

$$\varepsilon^A_y \sim N\left(0, \left(\sigma^A_r\right)^2\right) \quad , \qquad y = 1984, ..., y_{n-1}$$

The remaining estimable parameters are defined as having the near non-informative prior distributions:

$\ln\left(k^A_N\right) \sim U(-100, 0.7)$ (upper bound corresponding to $k^A_N = 2$)

$\ln\left(k^A_r\right) \sim U(-100, 0.7)$ (upper bound corresponding to $k^A_r = 2$)

$\ln\left(k^A_p\right) \sim U(-100, 0.7)$ (upper bound corresponding to $k^A_p = 2$)

$\left(\lambda^A_r\right)^2 \sim U(0, 100)$

$\left(\sigma^A_r\right)^2 \sim U(0, 10)$

$N_{1983,a}^{A} \sim U(0,500)$, $a = 0,1$

$N_{1983,a}^{A} \sim U(0,0.01)$, $a = 2,3$

$h^{A} \sim U(0.2,1.5)$

$\ln(K^{A}) \sim U(4.6,9.2)$ (corresponding to a range of about [100 000t; 1 000 000t] for K^{A})

$\eta_{y}^{ad} \sim N(0, \sigma_{ad}^{2})$

$\sigma_{ad} \sim U(0.1,0.5)$

$p \sim U(0,1)$

Further Outputs

Recruitment serial correlation:

$$s_{cor}^{A} = \frac{\sum_{y=1984}^{yn-2} \varepsilon_{y} \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{yn-2} \varepsilon_{y}^{2}\right)\left(\sum_{y=1984}^{yn-2} \varepsilon_{y+1}^{2}\right)}} \qquad \text{(A.9)}$$

and the standardised recruitment residual value for 2005:

$$\eta_{yn-1}^{A} = \frac{\varepsilon_{yn-1}^{A}}{\sigma_{r}^{A}}. \qquad \text{(A.10)}$$

are also required as input into the OMP.